

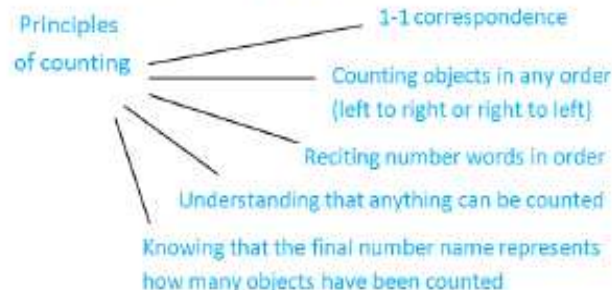
## Addition and Subtraction EYFS - KS1

### Early Years

**Mental strategies:**  
Recall number bonds to 10

#### Early Learning Goal EYFS

- **Mathematics Numbers:** Children count reliably with numbers 1-20, place them in order and say which number is one more or one less than a given number.
- Using quantities and objects, they add and subtract two single-digit numbers and count on or back to find the answer.



Subitising to 10 - When objects up to 10 are arranged in different ways children still recognise the quantity without counting e.g. children will see this as 2 sets of 4 and know the total is 8.



Unitising 10- Children need to understand how to regroup whenever they have 10 in preparation for place value.

E.g. 10 cubes make a rod/10 bricks make a house.

### Stage 1

**Mental strategies:**  
Recall number bonds to 20  
To know doubles to 20  
Add a single digit to 10

#### National Curriculum Expectations Year 1

- Read, write and interpret mathematical statements involving addition (+), subtraction (-) and equals (=) signs
- Represent and use number bonds and related subtraction facts within 20
- Add and subtract one-digit and two-digit numbers to 20, including zero
- Solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems such as  $7 = \square - 9$ .

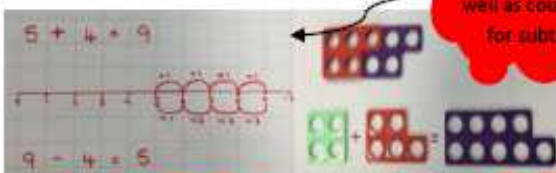
-Provide a variety of practical experiences for adding and subtracting using real life and mathematical resources.

-Number bonds should be explored in a range of representations to develop instant recall to 10/20



- Drawing can be used with numbers to 20 as a support.

-Number lines should be introduced alongside practical representations.



+ above number line  
- below number line

Teach vocabulary of 'ones' not units

Children need to be taught to count on to find the difference as well as counting back for subtraction

### Stage 2

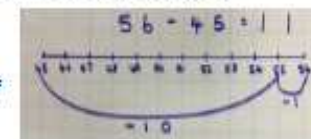
**Mental strategies:**  
Recall pairs of multiples of 10 which total 100  
Double multiples of 10 +/- multiples of 10 to any number up to 100

#### National Curriculum Expectations Year 2

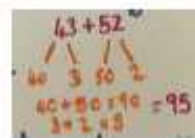
Solve problems with addition and subtraction:

- using concrete objects and pictorial representations, including those involving numbers, quantities and measures
- applying their increasing knowledge of mental and written methods
- Recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100
- Add and subtract numbers using concrete objects, pictorial representations, and mentally, including:
  - a two-digit number and ones/a two-digit number and tens/ two two-digit numbers
  - adding three one-digit numbers
- Show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot
- Recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems.

-Develop number lines- children need to be taught to use number lines efficiently by counting on and back in steps and using knowledge of number bonds.



-Children need to learn how to partition and recombine (any 2 digit number)



A range of practical resources must be used alongside calculations to develop understanding of place value.



-Bridging 100 e.g.  $74 + 28$

Teach vocabulary of 'ones' not units

#### Mastery Example NCETM

I'm thinking of a number. I've subtracted 5 and the answer is 7. What was the number I was thinking of. Explain how you know.

I'm thinking of a number. I've added 8 and the answer is 19. What number was I thinking of? Explain how you know?

All maths should be practical in EYFS unless children show evidence of mastery and are working at Greater depth. If introducing a number line, use alongside practical resources.

#### Mastery Example NCETM

Write the numbers 1-5 in each row so that each row and column adds up to the same number, called the 'magic number'. What is the magic number?



Children should not be moved on to working with numbers above 20 unless there is evidence of mastery and that they are working at greater depth.

#### Mastery Example NCETM

An odd number + an odd number + an odd number = an even number. Is this sometimes, always or never true? Explain your reasoning.

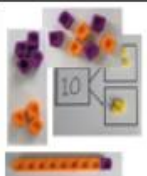



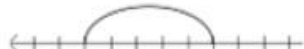


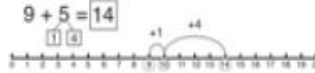


Children should not move on to larger numbers or different methods unless there is evidence of mastery and children are working at greater depth.

**Possible resources**







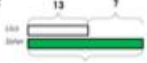

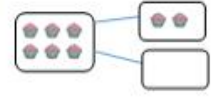
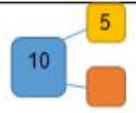
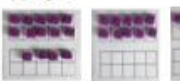

Numicon, Dienes, multi-link, counters, number lines, hundred squares, counting beads, abacus, place value counters, Lego, cars, dice, real world objects e.g. conkers, leaves, socks, gloves.



**Addition**

Concrete	Pictorial	Abstract
 <p>Use cubes to add two numbers together as a group or in a bar.</p>	 <p>Use pictures to add two numbers together as a group or in a bar.</p>	$4 + 3 = 7$  <p>Use the part-part whole diagram as shown above to move into the abstract.</p>
 <p>Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer.</p>	$12 + 5 = 17$  <p>Start at the larger number on the number line and count on in ones or in one jump to find the answer.</p>	$5 + 12 = 17$ <p>Place the larger number in your head and count on the smaller number to find your answer.</p>
$6 + 5 = 11$ <p>Start with the bigger number and use the smaller number to make 10.</p> 	 <p>Use pictures or a number line. Regroup or partition the smaller number to make 10.</p> $9 + 5 = 14$ 	$7 + 4 = 11$ <p>If I am at seven, how many more do I need to make 10. How many more do I add on now?</p>
$4 + 7 + 6 = 17$ <p>Put 4 and 6 together to make 10. Add on 7.</p> 	 <p>Add together three groups of objects. Draw a picture to recombine the groups to make 10.</p>	$4 + 7 + 6 = 10 + 7 = 17$ <p>Combine the two numbers that make 10 and then add on the remainder.</p>

**Subtraction**

Concrete	Pictorial	Abstract
<p>Use physical objects, counters, cubes etc to show how objects can be taken away.</p>  $6 - 2 = 4$	<p>Cross out drawn objects to show what has been taken away.</p>  $5 - 3 = 2$	$18 - 3 = 15$ $8 - 2 = 6$
<p>Make the larger number in your subtraction. Move the beads along your bead string or move counters as you count backwards in ones.</p> 	<p>Start at the bigger number and count back the smaller number showing the jumps on the number line.</p> 	<p>Put 13 in your head, count back 4. What number are you at? Use your fingers to help.</p>
<p>Compare amounts and objects to find the difference.</p>  <p>Use cubes to build towers or make bars to find the difference.</p>	<p>Count on to find the difference.</p>  <p>Draw bars to find the difference between 2 numbers.</p> 	<p>Hannah has 23 sandwiches, Helen has 15 sandwiches. Find the difference between the number of sandwiches.</p>
 <p>Link to addition- use the part whole model to help explain the inverse between addition and subtraction. If 10 is the whole and 6 is one of the parts. What is the other part? <math>10 - 6 =</math></p>	<p>Use a pictorial representation of objects to show the part part whole model.</p> 	 <p>Move to using numbers within the part whole model.</p>
$14 - 9 =$  <p>Make 14 on the ten frame. Take away the four first to make 10 and then take away one more so you have taken away 5. You are left with the answer of 9.</p>	$13 - 7 = 6$  <p>Start at 13. Take away 3 to reach 10. Then take away the remaining 4 so you have taken away 7 altogether. You have reached your answer.</p>	$16 - 8 =$ <p>How many do we take off to reach the next 10?          How many do we have left to take off?</p>

**Vocabulary**

add, more, make, sum, total, altogether, double, one more, two more ... ten more, How many more to make ...? How many more is ... than ...? How much more is ...? take away, How many are left/left over? How many have gone? one less, two less, ten less ... How many fewer is ... than ...? How much less is ...? difference between, digit equals, is the same as, number bonds/pairs missing number, number sentence, equals, symbol, method, operation, count on/count back, regroup, steps, mental method



# Addition and Subtraction LKS2

## Stage 3

National Curriculum Expectations Year 3

Add and subtract numbers mentally, including:

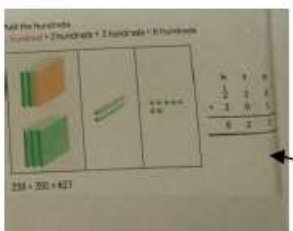
- A three-digit number and ones
- A three-digit number and tens
- A three-digit number and hundreds
- Add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction
- Estimate the answer to a calculation and use inverse operations to check answers
- Solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction.

### -Expanded column addition

### - Expanded decomposition

-Teach inverse to 'undo' calculation and check answers.

**Mental strategies**  
Recognise pairs of number which total 100 e.g. 32+68.  
Use number bond knowledge to find bonds to 1000 +/- near multiples of 10/100



Children must use resources to support their understanding.

Carrying and exchanging should be taught alongside no carrying/exchanging calculations so that children have to think about the numbers.

Hundreds/Tens/Ones (not units)

## Stage 4

National Curriculum Expectations Year 4

- Add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate
- Estimate and use inverse operations to check answers to a calculation
- Solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why.

**Mental strategies**  
Use knowledge of number bonds to find what must be added to decimals to make next whole number  
Apply place value and number bonds knowledge to larger numbers

### - Column Addition

Children should be extended to adding 3 two digit numbers or 3 three digit numbers as this is harder than bigger numbers.

### - Decomposition

Link to decimals in the context of money and measures. Number lines must be used for time and money. Children should also be introduced to rounding and estimating to check calculations.

### -Consolidate inverse to check calculations

Children need to learn when it is more efficient to count on and find the difference for subtraction

**Mastery Example NCETM**  
For positive integers are the following statements always, sometimes or never true.

- The sum of 2 odd numbers is even.
- The sum of 3 odd numbers is even.
- Adding 5 to a number ending in 6 will sum to a number ending in 1
- Adding 8 to a number ending in 2 will always sum to a multiple of 10.

Explain why in each case.

Children should not be moved on to working with numbers above 999 or other methods unless they show mastery and are working at greater depth.

**Mastery Example NCETM**  
Write 3 calculations where you would use mental calculation strategies and 3 where you apply a column method. Explain the decision you made for each calculation.

Children should be able to identify whether mental strategies could be used instead of column method. They should solve calculations with any 4 digit number and decimals to 1dp in any context with resources of their choice to demonstrate mastery.

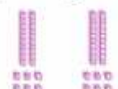
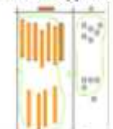
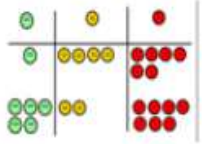
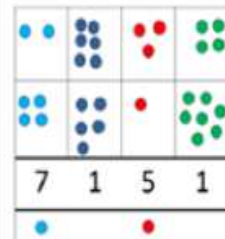


### Possible resources

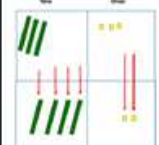
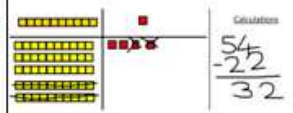
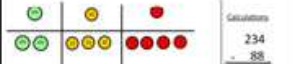
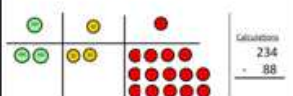
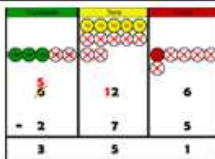

Numicon, Dienes, multi-link, counters, number lines, hundred squares, counting beads, abacus, place value counters, Lego, cars, dice, real world objects e.g. conkers, leaves, socks, gloves.



### Addition

Concrete	Pictorial	Abstract
<p>26 + 26</p> <p>Add together the ones first. Children regroup when they have 10. Then add the 10s.</p> 	<p>After practically using Dienes, children can draw them or use pictures of them to support calculation.</p> 	<p>Calculations</p> $\begin{array}{r} 21 \\ + 42 \\ \hline \end{array}$
<p>Once children are confident with Dienes they can be replaced by place value tokens.</p>  <p>Children should exchange tokens as they would with Dienes.</p>	<p>Children can draw a pictorial representation of the columns and place value tokens to further support their learning and understanding.</p> 	<p>Start by partitioning the numbers before moving on to clearly show the exchange below the addition.</p> $\begin{array}{r} 20 + 5 \\ 40 + 8 \\ 60 + 13 = 73 \end{array}$ <p>As the children move on, introduce decimals with the same number of decimal places and different. Money can be used here.</p> $\begin{array}{r} 536 \\ + 85 \\ \hline 621 \\ 11 \end{array}$

### Subtraction

Concrete	Pictorial	Abstract
<p>Use Dienes to make the bigger number then take the smaller number away. Teach children to exchange a ten for units when needed.</p> 	<p>Draw the Dienes alongside the written calculation to help to show working.</p> 	<p>Calculations</p> $\begin{array}{r} 47 - 24 = 23 \\ - 40 + 7 \\ - 20 + 6 \\ \hline 20 + 3 \end{array}$ <p>This will lead to a clear written column subtraction.</p> $\begin{array}{r} 32 \\ - 12 \\ \hline 20 \end{array}$
<p>Once children are confident with Dienes move to place value tokens.</p> <p>Make the larger number with the place value counters</p>  <p>Start with the ones, can I take away 8 from 4 easily? I need to exchange one of my tens for ten ones.</p>  <p>Now I can subtract my ones.</p> <p>Now look at the tens, can I take away 8 tens easily? I need to exchange one</p>	<p>Draw the counters onto a place value grid and show what you have taken away by crossing the counters out as well as clearly showing the exchanges you make.</p>  <p>When confident, children can find their own way to record the exchange/regrouping.</p>  <p>Just writing the numbers as shown here shows that the child understands the method and knows when to exchange/regroup.</p>	<p>Children can start their formal written method by partitioning the number into clear place value columns.</p> $\begin{array}{r} 836 - 254 = 582 \\ 800 \ 30 \ 6 \\ - 200 \ 50 \ 4 \\ \hline 500 \ 80 \ 2 \end{array}$ <p>Moving forward the children use a more compact method.</p> $\begin{array}{r} 836 - 254 = 582 \\ \begin{array}{r} 800 \\ + 30 \\ + 6 \end{array} \\ - \begin{array}{r} 200 \\ 50 \\ 4 \end{array} \\ \hline 500 \ 80 \ 2 \end{array}$ <p>This will lead to an understanding of subtracting any number</p>

### Vocabulary

equals, is the same as, number bonds/pairs missing number, number sentence, equals, symbol, method, operation, count on/count back, regroup, steps, mental method, number bonds/pairs/facts, tens boundary, partition, plus, minus, leave, place holder, efficient, inverse, increase, decrease, calculation, strategy, equation, balance, equivalent to, integer, decimal, represents, decimal



# Addition and Subtraction UKS2

## Stage 5

National Curriculum Expectations Year 5

- Add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction)
- Add and subtract numbers mentally with increasingly large numbers
- Use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy
- Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why

**Mental strategies**  
 Recall +/- facts for near multiples of 10  
 Use knowledge of near doubles and halves to quickly solve calculations

- Column Addition → Strengthening use/understanding of decimals
- Decomposition → Rounding and estimating to check calculations
- Explore negative numbers in the context of measures
- Strengthen use of number line and mental strategies to solve calculation efficiently.



### - Reasoning

Build on previous skills and develop understanding by giving children rich and varied problem solving tasks in a variety of contexts.

Children must use resources to support their understanding of increasingly complex problems



Hundreds/Tens/Ones (not units)

**Mastery Example NCETM**

True or False?

3999-2999=4000-3000  
 3999-2999=3000-2000  
 2741-1263=2742+1264  
 2741-1263=2731-1253  
 2741+1263=2742-1252

Explain your reasoning.

## Stage 6

National Curriculum Expectations Year 6

- Use their knowledge of the order of operations to carry out calculations involving the four operations
- Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why
- Solve problems involving addition, subtraction, multiplication and division
- Use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy.

Children should continue to build on the strategies they have learnt in more complex problems/contexts.

Children should always be encouraged to justify answers using mathematical vocabulary in order to challenge their understanding. They should be able to explain their choice of method ensuring that it is efficient.

**Mastery Example NCETM**

Use this number sentence to write down 3 more pairs of decimal numbers to sum 3.  
 $1.6 + 1.4 = 3$

If I keep subtracting 3 from 397 I will get to 0. Do you agree or disagree?  
 Explain your reasoning.

Hundreds/Tens/Ones (not units)

**Rich Example**

Kangaroo Subtraction  
 Each of the letters K, A, N, G, R, O represent a different digit.

$$\begin{array}{r} \text{KAN} \\ - \text{GAR} \\ \hline \text{00} \end{array}$$

What is the possible value of the number KAN?

$$\begin{array}{r} 32.8 \\ -20.68 \\ \hline 12.12 \end{array}$$

Children must demonstrate Mastery in a wide range of contexts in order to be working at greater depth. They should be able to justify and reason using a wide range of technical vocabulary.

**Mastery Example WhiteRose**

3 Here is a rule for generating a sequence.

Multiply the previous term by 3 and subtract 4

The second term of the sequence is 5

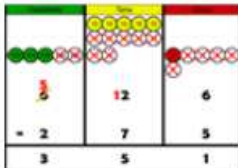

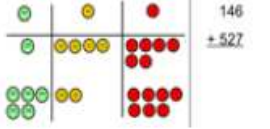
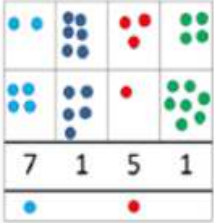
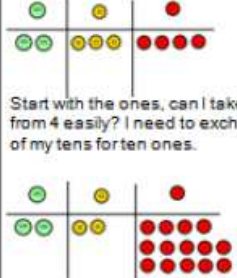
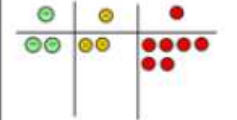
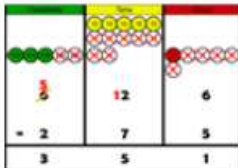


Find the difference between the first and fourth terms of the sequence.



### Possible resources

Numicon, Dienes, multi-link, counters, number lines, hundred squares, counting beads, abacus, place value counters, Lego, cars, dice, real world objects e.g. conkers, leaves, socks, gloves.



Concrete	Pictorial	Abstract	Once children are confident with Dienes move to place value tokens.		Draw the counters onto a place value grid and show what you have taken away by crossing the counters out as well as clearly showing the exchanges you make.	
<p>Once children are confident with Dienes they can be replaced by place value tokens.</p>  <p>Children should exchange tokens as they would with Dienes.</p>	<p>Children can draw a pictorial representation of the columns and place value tokens to further support their learning and understanding.</p> 	<p>Start by partitioning the numbers before moving on to clearly show the exchange below the addition.</p> $\begin{array}{r} 20 + 5 \\ 40 + 8 \\ 60 + 13 = 73 \end{array}$ <p>As the children move on, introduce decimals with the same number of decimal places and different. Money can be used here.</p> $\begin{array}{r} 536 \\ + 85 \\ \hline 621 \\ 11 \end{array}$ $\begin{array}{r} 72.8 \\ + 54.6 \\ \hline 127.4 \\ 11 \end{array}$	<p>Make the larger number with the place value counters</p>  <p>Start with the ones, can I take away 8 from 4 easily? I need to exchange one of my tens for ten ones.</p> <p>Now I can subtract my ones.</p>  <p>Now look at the tens, can I take away 8 tens easily? I need to exchange one hundred for ten tens.</p>	  <p>the child understands the method and knows when to exchange/regroup.</p>	<p>When confident, children can find their own way to record the exchange/regrouping.</p> <p>Just writing the numbers as shown here shows that</p>	<p>Children can start their formal written method by partitioning the number into clear place value columns.</p>  <p>Moving forward the children use a more compact method.</p> <p>This will lead to an understanding of subtracting any number including decimals.</p> $\begin{array}{r} 5 \ 12 \ 1 \\ - 2 \ 6 \ 5 \\ \hline 2 \ 6 \ 5 \end{array}$

### Vocabulary

Increase, decrease, calculation, strategy, equation, balance, equivalent to, integer, decimal, represents, decimal point, negative number, round, sequence, compare, strategy, reduce, boundary, relationship, consecutive, pattern, rule, logical

Early Years

**Mental strategies:**  
Double numbers to 10.  
Halve even numbers to 10

Early Learning Goal EYFS

Mathematics Numbers: Solve problems involving halving and doubling

Principles of counting

- 1-1 correspondence
- Counting objects in any order (left to right or right to left)
- Reciting number words in order
- Understanding that anything can be counted
- Knowing that the final number name represents how many objects have been counted

-Share, halve and double practically in real life contexts. e.g. food, toys, coins.



- Teach children to recognising the link between halving and doubling i.e. half of 4 is 2 so double 2 is 4.

- Unitising 10- Children need to understand how to regroup when they have 10.

-Encourage organising, reorganising and sorting objects in to lines, dice dots and arrays so that children can count efficiently and recognise amounts instantly without counting (subitising).



Multiplication and Division EYFS- KS1

Stage 1

**Mental strategies:**  
Count in 2s to 24, 5s to 60 and 10s to 120  
Double numbers to 20  
Halve even numbers to 20

National Curriculum Expectations Year 1

• Solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher

-Arrays- provide rich variety such as; printing, real life objects, pictures, peg boards and children's own pictorial representations.



-Develop unitising by encouraging children to regroup for 2, 5 and 10 and count in steps.

- Halving -strengthen and develop from EYFS using a range of practical resources. Discuss 'what happens when you halve an odd number?'

- Provide opportunities for children to share objects practically and using drawings.

"Sharing" should become "grouping" as children move from year 1 into year 2. The term 'sharing' should only be used in EYFS/Year 1.

-Double numbers to 10 with a range of resources to develop instant recall.



Stage 2

**Mental strategies:**  
Double any multiple of 10 to 100  
Halve multiples of 10 to 100  
Identify odd and even numbers to 100  
Count in steps of 2, 3, 5 and 10 forward and backward

National Curriculum Expectations Year 2

• Recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers  
• Calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (x), division (÷) and equals (=) signs

• Show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot  
• Solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in context  
- Arrays to be used as with year one but alongside calculation.

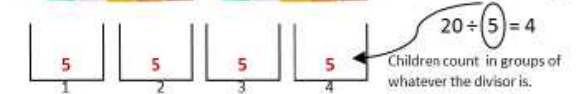
- X ÷ should be taught together so that children understand how they relate. Encourage children to find families of multiplication and division facts.



- Grouping- practical, pictorial and abstract methods to reinforce times tables knowledge when dividing. e.g. 20 ÷ 5 = 'How many groups of 5 can you make with 20?'



'Sweet and bag' method



- Multiplication and division should be taught on a number line alongside practical strategies and resources.



When teaching division on a number line use additive links as this leaves less room for error and reinforces times tables.

**Mastery Example NCETM**  
I can double any number but only halve some numbers. Do you agree? Explain your reasoning.

All maths should be practical in EYFS. Unless children show evidence of mastery and are working at greater depth, they should not formally record.

**Mastery Example NCETM**  
If I start at 0 and count on in fives will I say the number 55? If I start on 4 and count on in 2s will I say the number 17? If I start at 10 and count on in 10s will I say 100? Explain your reasoning.

Children should not be moved on to working with numbers above 20 unless there is evidence of mastery and they are working at greater depth.

**Mastery Example NCETM**  
True or false?  
5x4=4x5                      5x4=10x2                      5x4=2x10  
Explain your reasoning. What do you notice?





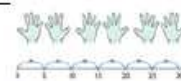





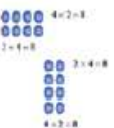

Children should not move beyond 2, 5 and 10 times tables unless there is evidence of mastery and they are working at greater depth.

**Possible resources**


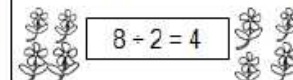
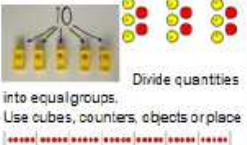





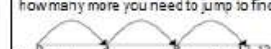
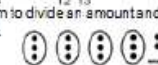

Numicon, Dienes, multi-link, counters, number lines, hundred squares, multiplication grids, counting beads, abacus, place value counters, cars, dice, real world objects e.g. conkers, leaves, socks, gloves.



**Multiplication**

Objective and Strategies	Concrete	Pictorial	Abstract
<b>Doubling</b>	Use practical activities to show how to double a number. 	Draw pictures to show how to double a number. Double 4 is 8 	Partition a number and then double each part before recombining it back together. 
<b>Counting in multiples</b>	Count in multiples supported by concrete objects in equal groups. 	Use a number line or pictures to continue support in counting in multiples. 	Count in multiples of a number aloud. Write sequences with multiples of numbers. 2, 4, 6, 8, 10 5, 10, 15, 20, 25, 30
<b>Repeated addition</b>	Use different objects to add equal groups. 	Write addition sentences to describe objects and pictures.  2 add 2 add 2 equals 6  5 + 5 + 5 = 15	Write addition sentences to describe objects and pictures.  2 + 2 + 2 + 2 + 2 = 10
<b>Arrays - showing commutative multiplication</b>	Create arrays using counters/cubes to show multiplication sentences. 	Draw arrays in different rotations to find commutative multiplication sentences. 	Use an array to write multiplication sentences and reinforce repeated addition.  5 + 5 + 5 = 15 3 + 3 + 3 + 3 + 3 = 15 3 x 5 = 15 5 x 3 = 15

**Division**

Objective and Strategies	Concrete	Pictorial	Abstract
<b>Sharing objects into groups</b>	I have 10 cubes, can you share them equally in 2 groups? 	Children use pictures or shapes to share quantities. 8 ÷ 2 = 4 	Share 9 buns between three people. 9 ÷ 3 = 3
<b>Division as grouping</b>	Divide quantities into equal groups. Use cubes, counters, objects or place value. 	Use a number line to show jumps in groups. The number of jumps equals the number of groups.  Think of the bars as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group.  20 ÷ 5 = 4 5 x 4 = 20	28 ÷ 7 = 4 Divide 28 into 7 groups. How many are in each group?
<b>Division within arrays</b>	Link division to multiplication by creating an array and thinking about the number sentences that can be created. Eg 15 ÷ 3 = 5    5 x 3 = 15 15 ÷ 5 = 3    3 x 5 = 15 	Draw an array and use lines to split the array into groups to make multiplication and division sentences. 	Find the inverse of multiplication and division sentences by creating four linking number sentences. 7 x 4 = 28 4 x 7 = 28 28 ÷ 7 = 4 28 ÷ 4 = 7
<b>Division with a remainder</b>	14 ÷ 3 = Divide objects between groups and see how much is left over. 	Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.  Draw dots and group them to divide an amount and clearly show a remainder. 	Complete written divisions and show the remainder using r. 20 ÷ 3 = 6 REMAINDER 2 

**Vocabulary**

Groups of, lots of, share, regroup, double, halve, multiple, repeated addition, array, regroup, factor, product, multiple, multiply, divide, method, strategy, remainder, calculation, symbol



# Multiplication and Division LKS2

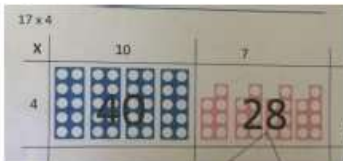
## Stage 3

National Curriculum Expectations Year 3

- Recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables
- Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods
- Solve problems, including missing number problems, involving multiplication and division, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects.

- Introduce grid method for multiplication alongside resources to aid children's understanding.

- 1 1 digit by 2 digit
- 2 1 digit by 3 digit 2 digit

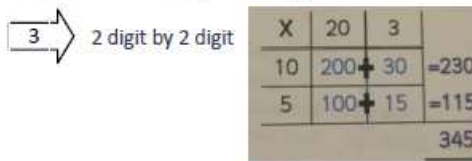


## Stage 4

National Curriculum Expectations Year 4

- Recall multiplication and division facts for multiplication tables up to  $12 \times 12$
- Use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together three numbers
- Recognise and use factor pairs and commutativity in mental calculations
- Multiply two-digit and three-digit numbers by a one-digit number using formal written layout
- Solve problems involving multiplying and adding, including using the distributive law to multiply two digit numbers by one digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects.

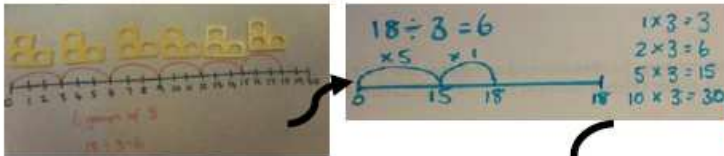
- Children to develop use of grid method.



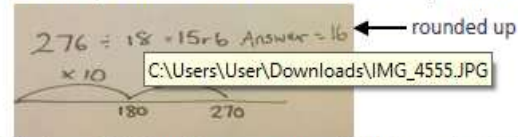
**Mental strategies**  
 Develop quick recall of all times tables facts  
 Begin to recall related division facts  
 Use knowledge of place value to multiply and divide any single digit number by a multiple of 10, 100 or 1000  
 Count in steps of 6, 7, 9, 25 and 100  
 Recognise and use factor pairs

Teach children when to use grid method and when to use times tables knowledge. Encourage mental methods for calculations that involve known multiplication facts.

- Division on number line- teach as additive as this leads to less errors and reinforces times tables. Fact boxes should be recorded alongside number line using layout shown below.



Use 1x, 2x, 5x and 10 times the divisor for initial fact boxes as this enables children to find most other related facts.



- Develop use of number lines for division. Children should explore calculations with remainders and round up and down in the contexts of problems.

- Teach expanded short division (bus stop method) for calculations with single digit divisors. Number lines should be used for 2/3 digit divisors.

Fact box  
 $1 \times 7 = 7$   
 $2 \times 7 = 14$   
 $5 \times 7 = 35$   
 $10 \times 7 = 70$

50 lots of 7  
 $7 \overline{) 350} \begin{matrix} 7 & 5 & \\ \underline{49} & & \\ 5 & & \\ \underline{35} & & \\ 5 & & \\ \underline{35} & & \\ 0 & & \end{matrix}$  Remainder 5

$362 \div 7 =$   
 $7 \overline{) 362} \begin{matrix} 5 & 1 & r5 \\ \underline{35} & & \\ 1 & 2 & \\ \underline{7} & & \\ 5 & & \\ \underline{35} & & \\ 2 & & \end{matrix}$   
 $362 \div 7 = 51 \text{ r}5$

Children can use resources or jottings to support their mental calculations if needed

$178 \div 12 = 14 \text{ r}10$   
 Fact Box  
 $12 \times 1 = 12$   
 $12 \times 2 = 24$   
 $12 \times 10 = 120$   
 $12 \times 4 = 48$

Teach children how to use multiplication square as a resources to support their calculations

**Mastery Example NCETM**  
 Sam is planting onions in his vegetable garden. He arranges the onions in to rows of 4 and has 2 left over. He then arranges them in to rows of 3 and has none left over. How many onions might he have had? Explain your reasoning.

**Mastery Example NCETM**  
 Multiply a number by itself and then make one factor one more and the other one less. What happened to the product.  
 E.g.  $4 \times 4$       $6 \times 6$   
 $5 \times 3$       $7 \times 5$   
 What do you notice? Will this always happen?

Children should use a range of resources to support calculations. They should be able to apply these methods in a range of contexts in order to demonstrate mastery.

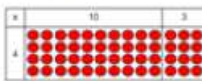
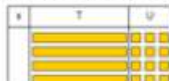
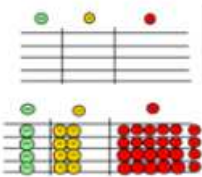
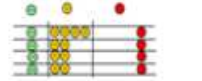

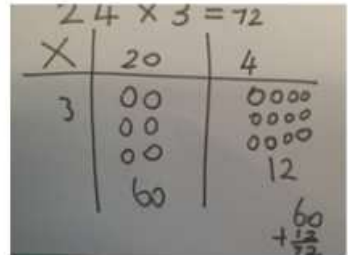
Children should be presented with problems in a range of contexts to deepen their understanding and evidence mastery.

**Possible resources**

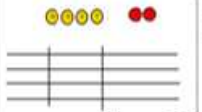
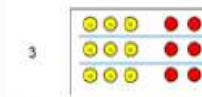
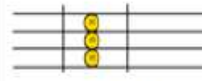
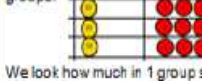
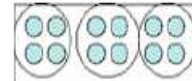
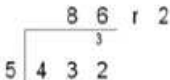
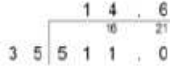
Numicon, Dienes, multi-link, counters, number lines, hundred squares, multiplication grids, counting beads, abacus, place value counters, cars, dice, real world objects e.g. conkers, leaves, socks, gloves.



**Multiplication**

Concrete	Pictorial	Abstract																														
<p>Show the link with arrays to first introduce the grid method.</p>  <p>4 rows of 10 4 rows of 3</p> <p>Move on to using Dienes to move towards a more compact method.</p>  <p>4 rows of 13</p> <p>Move on to place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows.</p>  <p>Calculate <math>4 \times 126</math></p>  <p>Fill each row with 126. Add up each column, starting with the ones making any exchanges needed.</p> 	<p>Children can represent the work they have done with Dienes or place value counters in a way that they understand.</p> <p>They can draw the counters, using colours to show different amounts or just use circles in the different columns to show their thinking as shown below.</p> 	<p>Start with multiplying by one digit numbers and showing the clear addition alongside the grid.</p> <table border="1" data-bbox="851 566 1030 622"> <tr><td>x</td><td>30</td><td>5</td></tr> <tr><td>7</td><td>210</td><td>35</td></tr> </table> <p><math>210 + 35 = 245</math></p> <p>Moving forward, multiply by a 2 digit number showing the different rows within the grid method.</p> <table border="1" data-bbox="851 805 1041 925"> <tr><td></td><td>10</td><td>8</td></tr> <tr><td>10</td><td>100</td><td>80</td></tr> <tr><td>3</td><td>30</td><td>24</td></tr> </table> <table border="1" data-bbox="851 949 1030 1037"> <tr><td>x</td><td>1000</td><td>300</td><td>40</td><td>2</td></tr> <tr><td>10</td><td>10000</td><td>3000</td><td>400</td><td>20</td></tr> <tr><td>8</td><td>8000</td><td>2400</td><td>320</td><td>16</td></tr> </table>	x	30	5	7	210	35		10	8	10	100	80	3	30	24	x	1000	300	40	2	10	10000	3000	400	20	8	8000	2400	320	16
x	30	5																														
7	210	35																														
	10	8																														
10	100	80																														
3	30	24																														
x	1000	300	40	2																												
10	10000	3000	400	20																												
8	8000	2400	320	16																												

**Division**

Objective and Strategies	Concrete	Pictorial	Abstract
<p>Short division</p>	<p>Use place value counters to divide</p>  <p>Calculate <math>42 \div 3</math></p>  <p>using the bus stop method alongside <math>42 \div 3 =</math></p> <p>Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.</p>  <p>We exchange this ten for ten ones and then share the ones equally among the groups.</p>  <p>We look how much in 1 group so the answer is 14.</p>	<p>Children can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.</p>  <p>Encourage them to move towards counting in multiples to divide more efficiently.</p>	<p>Finally move into decimal places to divide the total accurately.</p>  

**Vocabulary**

factor, product, multiple, groups of, lots of, multiply, divide, quotient, array, method, strategy, remainder, factor, product, multiple, groups of, lots of, multiply, divide, quotient, array, method, strategy, remainder, short division, long division, round

## Multiplication and Division UKS2

### Stage 5

#### National Curriculum Expectations Year 5

- Identify multiples and factors, including finding all factor pairs of a number, and common factors of 2 numbers
- Know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers
- Establish whether a number up to 100 is prime and recall prime numbers up to 19
- Multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers
- Multiply and divide numbers mentally, drawing upon known facts
- Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context
- Multiply and divide whole numbers and those involving decimals by 10, 100 and 1,000
- Recognise and use square numbers and cube numbers, and the notation for squared ( $^2$ ) and cubed ( $^3$ )
- Solve problems involving multiplication and division, including using their knowledge of factors and multiples, squares and cubes
- Solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign
- Solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates

-Consolidate children's use of grid method.



75 x 429 =		400	20	9
70	28,000	1400	630	
9	2000	100	45	

$$75 \times 429 = 28,000 + 1400 + 630$$

$$75 \times 429 = 32,175$$

- Short multiplication for efficiency

$$\begin{array}{r} 237 \\ \times 4 \\ \hline 948 \\ 1 \end{array}$$

Expanded vertical multiplication is optional. It is not necessary for all.

$$\begin{array}{r} 36 \\ \times 27 \\ \hline 42 \quad (7 \times 6) \\ 216 \quad (7 \times 30) \\ 720 \quad (20 \times 6) \\ 600 \quad (20 \times 30) \\ \hline 972 \end{array}$$

-Short division- (Single digit divisors) Teach children to convert remainders to decimals/fractions.

$$142 \div 4 =$$

$\begin{array}{r} 035 \cdot 5 \\ 4 \overline{) 142 \cdot 0} \end{array}$	<b>Fact Box</b> $2 \times 4 = 8$ $5 \times 4 = 20$ $10 \times 4 = 40$
--	--

$$= 35 \frac{2}{4} = 35 \frac{1}{2} = 35.5$$

- Consolidate additive chunking (2/3 digit divisors)

#### Mastery Example NCETM

Factors come in pairs so all numbers have an even number of factors. Do you agree? Explain your reasoning.

Children should use a range of resources to support calculations. They should be able to apply these methods in a range of contexts in order to demonstrate mastery.

#### Mental strategies

Double and halve decimals to 1DP  
Instantly recall multiplication and division facts up to  $12 \times 12$   
Know square numbers up to  $12 \times 12$   
Count forwards and backwards in powers of 10

### Stage 6

#### National Curriculum Expectations Year 6

- Multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication
- Divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context
- Divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context
- Perform mental calculations, including with mixed operations and large numbers
- Identify common factors, common multiples and prime numbers
- Use their knowledge of the order of operations to carry out calculations involving the four operations
- Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why
- Solve problems involving addition, subtraction, multiplication and division
- Use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy.

- Consolidate Short Multiplication methods

$$5.9 \times 3 =$$

x	5	0.9
3	15	2.7

$$5.9 \times 3 = 15 + 2.7$$

$$5.9 \times 3 = 17.7$$

Method 1

$$39 + 51 =$$


Method 2

Comparing strategies with children strengthens their understanding.

- Consolidate short division (single digit divisor) and additive chunking (2/3 digit divisor)

$$6 \overline{) 9941.00}$$

$$257 \div 17 = 15r2$$

Fact Box

$$\begin{array}{l} 17 \times 1 = 17 \\ 17 \times 2 = 34 \\ 17 \times 3 = 51 \\ 17 \times 4 = 68 \\ 17 \times 5 = 85 \\ 17 \times 6 = 102 \\ 17 \times 7 = 119 \\ 17 \times 8 = 136 \\ 17 \times 9 = 153 \\ 17 \times 10 = 170 \\ 17 \times 11 = 187 \\ 17 \times 12 = 204 \\ 17 \times 13 = 221 \\ 17 \times 14 = 238 \\ 17 \times 15 = 255 \end{array}$$

Teach children to develop efficient fact boxes.

-Brackets BIDMAS

Prime numbers are harder so ensure that children are presented with problems that include them.

#### Mastery Example NCETM

Which calculation is the odd one out? Explain your reasoning.  
 $753 \times 1.8 / (75.3 \times 3) \times 6 / 753 + 753 \div 5 \times 4 / 7.53 \times 1800 / 753 \times 2 - 753 \times 0.2 / 750 \times 1.8 + 3 \times 1.8$

Children should be presented with problems in a range of contexts to deepen their understanding and develop mastery of skills.

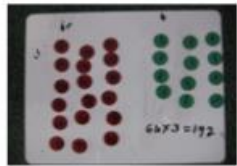
**Possible resources**

Numicon, Dienes, multi-link, counters, number lines, hundred squares, multiplication grids, counting beads, abacus, place value counters, cars, dice, real world objects e.g. conkers, leaves, socks, gloves.



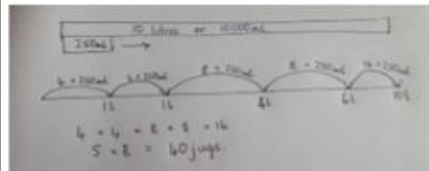
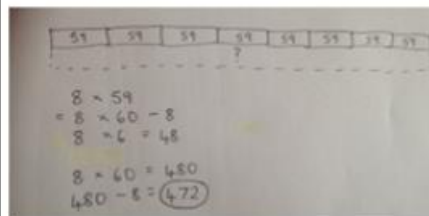
**Multiplication**

Children can continue to be supported by place value counters at the stage of multiplication.



It is important at this stage that they always multiply the ones first and note down their answer followed by the tens which they note below.

Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods.



Start with long multiplication, reminding the children about lining up their numbers clearly in columns.

If it helps, children can write out what they are solving next to their answer.

$$\begin{array}{r} 32 \\ \times 24 \\ \hline 120 \\ 640 \\ \hline 768 \end{array}$$

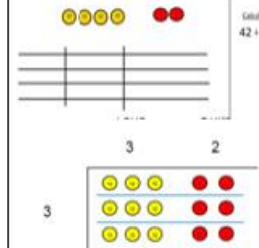
This moves to the more compact method.

$$\begin{array}{r} 32 \\ \times 18 \\ \hline 24 \\ 240 \\ \hline 576 \end{array}$$

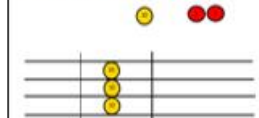
**Division**

**Concrete**

Use place value counters to divide



using the bus stop method alongside  $42 \div 3 =$   
Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.



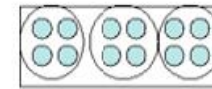
We exchange this ten for ten ones and then share the ones equally among the groups.



We look how much in 1 group so the answer is 14.

**Pictorial**

Children can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.



Encourage them to move towards counting in multiples to divide more efficiently.

**Abstract**

$$\begin{array}{r} 14 \text{ r } 2 \\ 3 \overline{)42} \\ \underline{30} \phantom{0} \\ 12 \phantom{0} \\ \underline{12} \phantom{0} \\ 0 \phantom{0} \end{array}$$

Finally move into decimal places to divide the total accurately.

$$\begin{array}{r} 14.6 \\ 3 \overline{)42.6} \\ \underline{30} \phantom{0} \\ 12 \phantom{0} \\ \underline{12} \phantom{0} \\ 0 \phantom{0} \end{array}$$

**Vocabulary**

factor, product, multiple, groups of, lots of, multiply, divide, quotient, array, method, strategy, remainder, short division, long division, round, factor, product, round, convert, decimal, fraction, percentage